AP Physics C – Calculus Based

This is your summer assignment for AP Physics C for scientists and engineers. Your summer assignment consists of an overview of vectors and 1-D and 2-D kinematics. Note that there will be calculus involved and which will include the derivative for which I have attached a primer in the appendix for those that are still to cover it.

It is expected that you review the pertinent materials covering a summary of vectors and kinematics. You are then to finish the attached multiple-choice problems and the one free response question.

This is a college level course and you are expected to use the resources at your disposal to figure out and learn how to solve the given problems.

If you need any help, please reference youtube and specifically AP Plus Physics (Dan Fullerton) online.

Enjoy the Summer.

Mr. Robayo

RHS STEM/Physics

Graphing

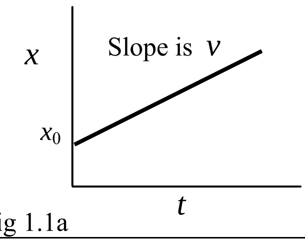
<table>
<tr><th></th><th>Displacement–Time Graph</th><th>Velocity–Time Graph</th><th>Acceleration–Time Graph</th></tr>
<tr><td>Constant Velocity</td><td>

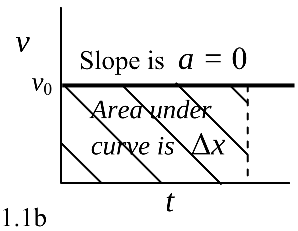
Fig 1.1a</td><td>

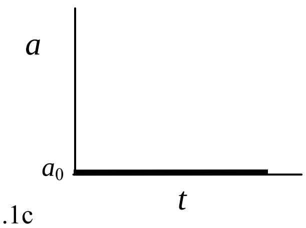
Fig 1.1b</td><td>

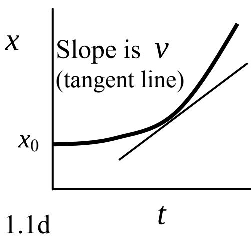
Fig 1.1c</td></tr>
<tr><td>Uniform Acceleration</td><td>

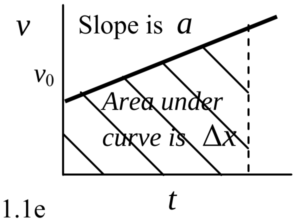
Fig 1.1d</td><td>

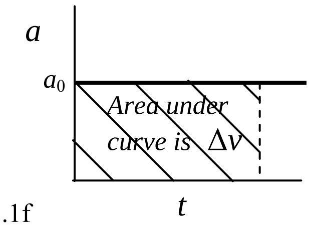
Fig 1.1e</td><td>

Fig 1.1f</td></tr>
</table>

Slopes of Curves are an important analytical tool used in physics. Any equation that can be manipulated into the format $y = mx + b$ can be represented and analyzed graphically. As an example: $v = v_0 + at$ can be rearranged slightly into $v = at + v_0$. Compare this equation to the equation of a line. It is apparent that a is the slope and that v_0 is the y-intercept (Fig 1.1b and Fig 1.1e). What equation generates velocity in Fig. 1.1a and Fig 1.1d?

Area Under a Curve is another important graphical tool. Multiply the y-axis (height) by the x-axis (base) and determine if this matches any known equations. For example: Figures 1.1b and 1.1e are velocity-time plots. Simply multiply $v \times t$. This is a rearranged form of the equation $v = x/t$. The form obtained from the graph is $x = vt$, which means that displacement is the area under the velocity-time plot.

Calculus: ***Required in AP Physics C*** (optional for AP Physics B students)***. Calculus is taught in math class. These review sheets will focus on the Physics aspect of solutions. Calculus steps may not be shown. Solution will be up to the student.***

The equations outlined in the previous page work well in the following situations.

- Linear Functions: involving constant velocity and acceleration, as diagrammed in the above graphs (except Fig 1.1d).
- Nonlinear Functions: scenarios where the problem is seeking information about a change in a quantity, Δx or Δv.
- Nonlinear Functions: scenarios where the problem is seeking an average velocity in an interval.

Calculus is needed to find the slopes of nonlinear functions and the areas under nonlinear curves.

1. **Velocity: Slope of the displacement-time curve.**

$v = \frac{dx}{dt}$ example: $v = \frac{d}{dt}\left(x_0 + v_0 t + \frac{1}{2}at^2\right) = v_0 + at$ $v = v_0 + at$

2. **Acceleration: Slope of the velocity-time curve.**

$a = \frac{dv}{dt}$ example: $a = \frac{d}{dt}\left(v_0 + at\right) = a$

3. **Velocity: Area under acceleration-time curve.** (Note: if $c = v_0$ cannot be found, then you can only solve for Δv)

$v = \int a \ dt$ example: $v = \int (a) \ dt = c + at$ $v = v_0 + at$

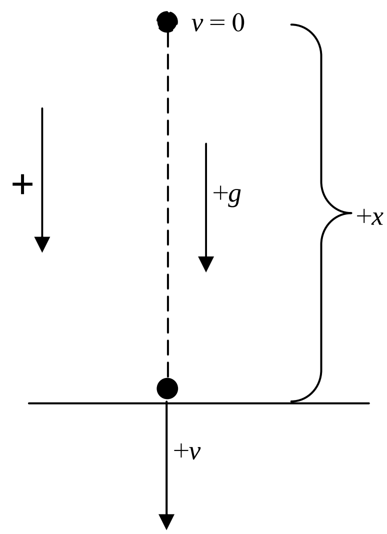
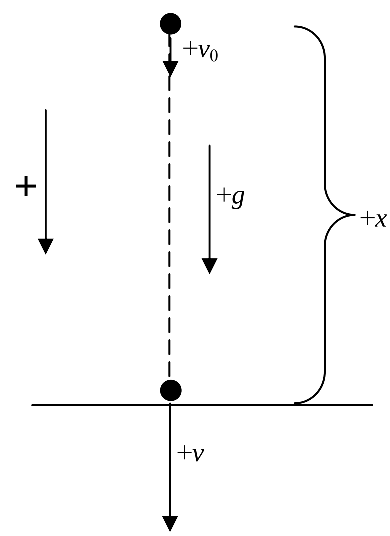
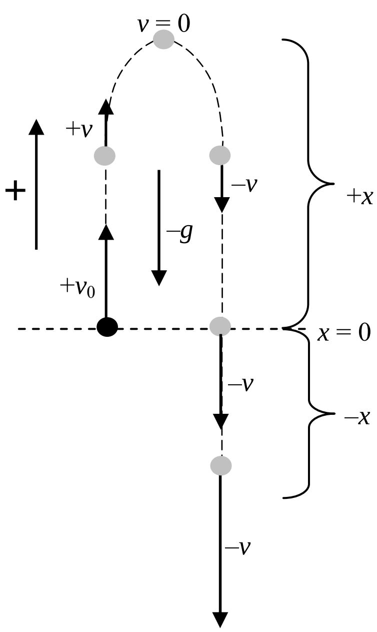
4. **Displacement: Area under the velocity-time curve.** (Note: if $c = x_0$ cannot be found, then you can only solve Δx)

$x = \int v \ dt$ example: $x = \int (v_0 + at) \ dt = c + v_0 t + \frac{1}{2}at^2$ $x = x_0 + v_0 t + \frac{1}{2}at^2$

Falling Bodies: Objects moving vertically under the influence of gravity. Earth's surface gravity is $g = -9.8$ m/s^2. This speeds objects up (+ acceleration), but is directed downward (–). Objects can be thrown up, down, or just be dropped. They can land below, at the same height, or above the origin. They come to an instantaneous stop at the highest point in flight.

$$\boxed{v_y = v_{0y} + gt} \qquad \boxed{y = y_0 + v_{0y}t + \frac{1}{2}gt^2} \qquad \boxed{{v_y}^2 = {v_{0y}}^2 + 2g\left(y - y_0\right)}$$

The positive and negative signs can cause trouble in these problems. The easiest way to handle the signs is set the direction of initial motion as positive and then to ensure all signs are consistent with this decision. This has one huge benefit. It eliminated the double sign on acceleration. When initial velocity v_0 is used to anchor direction, then a positive acceleration means speeding up and negative acceleration involves slowing down.

Dropped from rest	**Thrown downward**	**Thrown upward**
$v_0 = 0$, but it will move down initially	v_0 is directed downward	v_0 is directed upward
Set downward as positive direction	Set downward as positive direction	Set upward as positive direction
		
Everything is positive and easy	Everything is positive and easy	This is difficult, and depends on where in the flight the problem ends.

Kinematics Problems Involving Changes in the Magnitude of Acceleration

If the magnitude of acceleration changes while solving a kinematics problem then the problem must be solved in separate parts. Unlike displacement x and velocity v, the Kinematic equations do not contain the variables a_0 and a.

Example 1-3: More than one acceleration

A car initially at rest accelerates at 4 m/s^2 while covering a distance of 100 m. Then the car continues at constant velocity for 500 m. Finally it slows to a stop with a deceleration of 3 m/s^2. Determine the total time of this displacement.

Acceleration Phase: $x = \frac{1}{2}at^2 \qquad t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2(100)}{(4)}} = 7.07\,s$ and $v = v_0 + at = (0) + (4)(7.07) = 28.3\,m/s$

Constant Velocity Phase: $v = \frac{\Delta x}{t} \qquad t = \frac{\Delta x}{v} = \frac{(500)}{(28.3)} = 17.7\,s \qquad t = \frac{500}{v}$

Deceleration Phase: $v = v_0 + at \qquad t = \frac{v - v_0}{a} = \frac{(0) - (28.3)}{(-3)} = 9.43\,s$

Total Time = 7.07 + 17.7 + 9.43 = 34.2 s

1–2 Vectors and Vectors in Kinematics

Scalar: A physical quantity described by a single number and units. A quantity describing ***magnitude*** only.

Vector: Many of the variables in physics equations are vector quantities. Vectors have ***magnitude*** and ***direction***.

Magnitude: Size or extend. The numerical value.

Direction: Alignment or orientation with respect to set location and system of orientation, such as a coordinate axis.

Notation: $\vec{A}$ or $\xrightarrow{A}$ The length of the arrow represents, and is proportional to, the vectors magnitude. The direction the arrow points indicates the direction of the vector.

Negative Vectors: Have the same magnitude as their positive counterpart, but point in the opposite direction.

If $\xrightarrow{A}$ then $\xleftarrow{-A}$

Vector Addition and subtraction

Think of it as vector addition only. The result of adding vectors is called the **<u>result</u>**ant $\vec{R}$.

$\vec{A}+\vec{B}=\vec{R}$ $\xrightarrow{A}$ + $\xrightarrow{B}$ = $\xrightarrow{R}$

When you need to subtract one vector from another think of the one being subtracted as being a negative vector.

$\vec{A}-\vec{B}$ is really $\vec{A}+\left(-\vec{B}\right)=\vec{R}$ $\xrightarrow{A}$ + $\xleftarrow{(-B)}$ = $\xrightarrow{R}$

A negative vector has the same length as its positive counterpart, but its direction is reversed. ***<u>This is very important</u>***. In physics a negative number does not always mean a smaller number. Mathematically –2 is smaller than +2, but in physics these numbers have the same magnitude (size), they just point in different directions (180^o apart).

Vector Addition: Parallelogram

A + ***B***

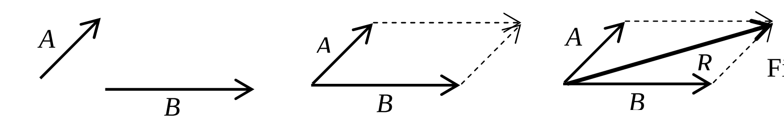


Fig 2.1a

Tip to Tail

A + ***B***

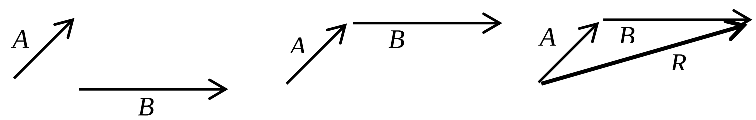


Fig 2.1b

Both methods arrive at the exact same solution since either method is essentially a parallelogram. In some problems one method is advantageous, while in other problems the alternative method is superior.

Reporting Magnitude and Direction:

Component Method: A vector a can be reported by giving the components along the *x*- or *y*-axis. Reporting a vector this way is formally done by employing the unit vectors **i** and **j**. As an example: vector A in fig 2.2a would be $\boxed{A = A_x\mathbf{i} + A_y\mathbf{j}}$, where, $|\mathbf{i}|+|\mathbf{j}|=\mathbf{1}$. There is at third component vector **k** used for three dimensional problems involving the *z*-axis. The vector in Fig. 2.2b shows a numerical application of the component method. In this example you are given the polar coordinates $A = 5\ at\ 37^o$. Using trigonometry the components can be established.

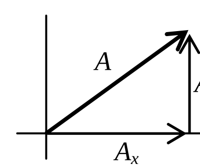


Fig. 2.2a

$\boxed{A_x = A\cos\theta} = 5\cos 37^o = 4$ and $\boxed{A_y = A\sin\theta} = 5\sin 37^o = 3$. Then A can be expressed as follows: $A = 4\mathbf{i} + 3\mathbf{j}$

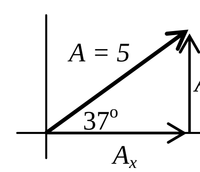


Fig. 2.2b

Polar Coordinates: Vector A in Fig. 2.2b is reported in polar coordinates, $A = 5\ at\ 37^o$. This is simply the length of a vector and its angle measured counterclockwise with respect to the positive *x*-axis. (Negative angles are allowed and indicate that direction was measured clockwise from the +*x*-axis. If the component vectors are given, $A = 4\mathbf{i} + 3\mathbf{j}$, Pythagorean theorem is used to establish the length of the parent vector $\boxed{A = \sqrt{A_x^{\ 2} + A_y^{\ 2}}} = \sqrt{4^2+3^2} = 5$.

Arctangent is used to find the direction $\boxed{\theta = \tan^{-1}\frac{A_y}{A_x}} = \tan^{-1}\frac{3}{4} = 37^o$. ***But, watch out! The angle arrived at by the arctangent formula may not be the final answer. The quadrant that the final vector lies in must be established, and an adjustment to the angle may be needed in order to provide an answer that extends from the +x-axis.***

Component Advantage In Vector Addition:

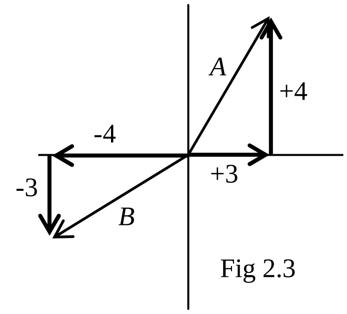


Fig 2.3

1. If vector components along an axis are used, direction can be specified with + and – symbols. Vectors A and B in Fig 2.3 have been converted into components.

   $A = (+3)\,\mathbf{i} + (+4)\,\mathbf{j}$ $\qquad$ $B = (-4)\,\mathbf{i} + (-3)\,\mathbf{j}$

2. This becomes advantageous if vectors A and B need to be added.

   Find the resultant of the x vectors: $\boxed{R_x = A_x + B_x} = (+3) + (-4) = -1$

   Find the resultant of the y vectors: $\boxed{R_y = A_y + B_y} = (+4) + (-3) = +1$

   Then combine them to find R: $\boxed{R = \sqrt{R_x^{\,2} + R_y^{\,2}}} = \sqrt{(-1)^2 + (+1)^2} = 1.41$

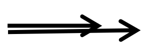
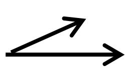
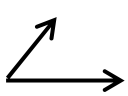
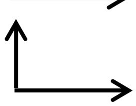
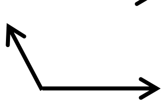
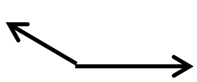
   Find the direction: $\boxed{\theta = \tan^{-1}\dfrac{R_y}{R_x}} = \tan^{-1}\dfrac{(+1)}{(-1)} = -45^o$, but this a 2nd quadrant angle and must be adjusted: $180^o - 45^o = 135^o$. The final vector is has a magnitude of 1.41 and a direction of 135^o.

3. Finding the components simplify problems throughout physics. In Newtonian Mechanics motion, force, and momentum often act at an angle to the *x*- or *y*-axis. Fortunately these vector quantities can be resolved into component vectors along the *x*- or *y*-axis. In addition, equations for motion, force, and momentum can be calculated in the *x*-direction independent of what is happening in the *y*-direction, and vice versa. The first example of this will be projectile motion. In projectile motion there will be a variety of initial and final of velocities at angles.

Scalar (Dot) Product of Two Vectors: The dot product (**A·B**) of two vectors **A** and **B** is a scalar and is equal to *AB*cosθ. This quantity shows how two vectors interact depending on how close to parallel the two vectors are. The magnitude of this scalar is largest when $\theta = 0^o$ (parallel) and when $\theta = 180^o$ (anti-parallel). The scalar is zero when $\theta = 90^o$ (perpendicular).

Vector (Cross) Product: The cross product (**A**×**B**) of two vectors **A** and **B** is a third vector **C**. The magnitude of this vector is $C = AB\sin\theta$. The direction of this vector is determined by the right-hand-rule. The order that the vectors are multiplied is important (not commutative) If you change the order of multiplication you must change the sign, **A**×**B = – A**×**B**. The magnitude of this vector is largest when $\theta = 90^o$ (perpendicular). The vector is zero when $\theta = 0^o$ (parallel) or when $\theta = 180^o$ (anti-parallel).

Examples: In the following examples vector **A** = 3 and **B** = 2. The direction of vector **A** will vary.

Vectors A & B	Scalar (Dot) Product	Vector (Cross Product)
	$(3)(2)\cos 0^o = \boxed{6}$	$(3)(2)\sin 0^o = \boxed{0}$
	$(3)(2)\cos 30^o = \boxed{5.2}$	$(3)(2)\sin 30^o = \boxed{3.0}$
	$(3)(2)\cos 60^o = \boxed{3}$	$(3)(2)\sin 60^o = \boxed{5.2}$
	$(3)(2)\cos 90^o = \boxed{0}$	$(3)(2)\sin 90^o = \boxed{6}$
	$(3)(2)\cos 120^o = \boxed{-3}$	$(3)(2)\sin 120^o = \boxed{5.2}$
	$(3)(2)\cos 150^o = \boxed{-5.2}$	$(3)(2)\sin 150^o = \boxed{3}$
	$(3)(2)\cos 180^o = \boxed{-6}$	$(3)(2)\sin 180^o = \boxed{0}$

Another way: When cos (parallel) appears in a formula you need two vectors that are parallel (+) or anti-parallel (–). Just use your trig skills to find a component of one of the vectors that points in the same direction as the other vector. In the examples above, find the component of vector **B** that is parallel to vector **A**. When sin (perpendicular) appears in a formula, find the component of vector **B** that is perpendicular to vector **A**.

1–3 Motion in Two Dimensions

Relative Velocity: Motion in two dimensions, both at constant velocity. A common example is that of a boat crossing a river. In figure 3.1, a boat leaves perpendicular to the shore with a velocity of v_B. The rivers current, v_R, carries it downstream a distance y. In figure 3.2 the boat aims at an angle of θ upstream in order to end up straight across. There is a triangle formed by the solid velocity vectors, and another formed by the dashed displacement vectors. These triangles are similar triangles. The resultant velocity of the boat will be a combination of the boats own velocity and current. This vector is labeled Σv. This is how fast the boat will appear to be moving as seen from a stationary observation point on shore. When calculating the time to cross the stream use the velocity vector and displacement vector that point in the same direction. In figure 3.1, $v_B = \frac{x}{t}$. In figure 3.2, $\sum v = \frac{x}{t}$

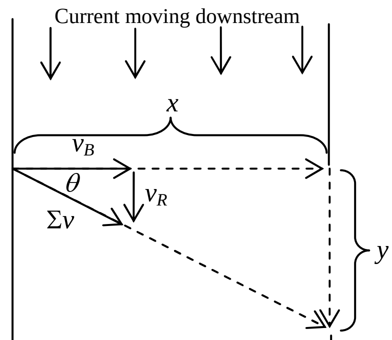


Fig 3.1

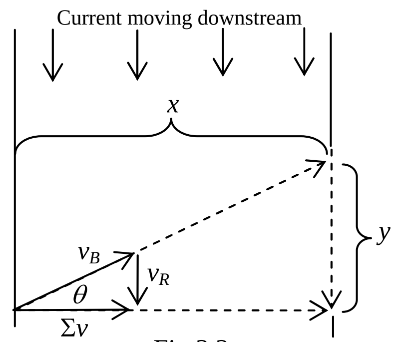


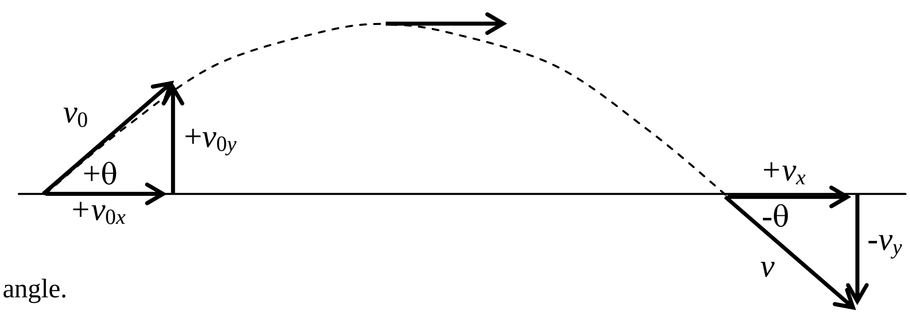
Fig 3.2

Projectile Motion: Motion in one dimension involves acceleration, while the other is at constant velocity.

- In the *x*-direction the velocity is constant, with no acceleration occurring in this dimension.
- In the *y*-direction the acceleration of gravity slows upward motion and enhances downward motion.

Vector Components in Projectile Motion: **The *x*-direction and the *y*-direction are independent of each other.**

Fig 3.3



Now v_0 is at an angle.

Solve for v_{0x}: $\boxed{v_{0x} = v_0 \cos\theta}$ then use v_{0x} in the kinematic equations to solve for v_x.

Solve for v_{0y}: $\boxed{v_{0y} = v_0 \sin\theta}$ then use v_{0y} in the falling body equations to solve for v_y.

v_x and v_y are component vectors. To find v, use Pythagorean Theorem $\boxed{v = \sqrt{v_x^{\,2} + v_y^{\,2}}}$ and arctangent $\boxed{\theta = \tan^{-1}\frac{v_y}{v_x}}$

The highest point in the flight: $\boxed{v_x = v_{0x}}$ and $\boxed{v_y = 0}$. If the problem ended here these conditions would apply.

Horizontal Launches

The launch angle is $\boxed{\theta = 0^o}$

$$v_{0x} = v_0 \cos\theta = v_0 \cos 0^o = v_0 \qquad \boxed{v_{0x} = v_0}$$

$$v_{0y} = v_0 \sin\theta = v_0 \sin 0^o = 0 \qquad \boxed{v_{0y} = 0}$$

The above math is not really necessary. Inspection of the Fig 3.4 shows that v_0 is directed straight down the *x*-axis with no *y*-component vector visible at all. The end of the problem is similar to the problem depicted in Fig 3.3, above.

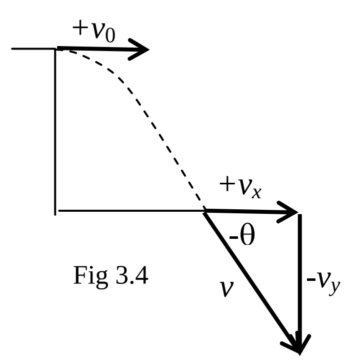


Fig 3.4

Coordinate Axis System provides the necessary orientation to handle the following variables and their appropriate signs: launch angle, initial velocities in x & y, final velocities in x & y, final landing height, and final overall velocity. Orientation matters and thus the coordinate axis becomes a powerful tool, as depicted on the next page.

Initial Launch

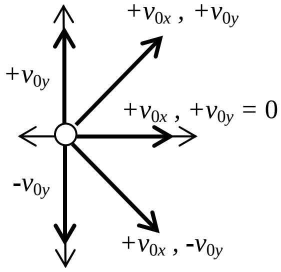


Fig 3.5

Initial displacement

$x_0 = 0$

$y_0 = 0$

Falling bodies: θ = ±90°

$v_{0x} = v_0 \cos 90^o = 0$

$v_{0y} = v_0 \sin 90^o = v_0$

Horizontal launch: θ = 0°

$v_{0x} = v_0 \cos 0^o = v_0$

$v_{0y} = v_0 \sin 0^o = 0$

1st quadrant launch: +θ

$v_{0x} = v_0 \cos\theta$ will be +

$v_{0y} = v_0 \sin\theta$ will be +

4th quadrant launch –θ

$v_{0x} = v_0 \cos\theta$ will be +

$v_{0y} = v_0 \sin\theta$ will be –

During the problem

(At the top $v_x = v_{ox}$ and $v_y = 0$)

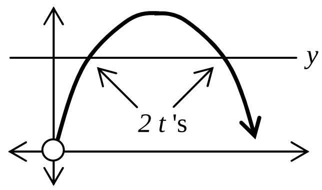


Fig 3.6

If it lands at the same height as it started ($y = y_0$), then $t_{up} = t_{down}$.

There are two t's for every y. The shorter t is for the upward trip. The longer t is for the downward trip.

Solve for maximum height two ways

1. From ground up where $v_y = 0$.
   $v_y^{\ 2} = v_{0y}^{\ 2} + 2g\left(y - y_0\right)$
2. Or the easy way. Start at the top and pretend it is a falling body. v_x doesn't matter since time is controlled by the y-direction. And at the top v_{oy} is zero. However, this solves for half of the total flight.
   $y = \frac{1}{2}gt^2$ ***<u>Must double time!</u>***

Final Landing

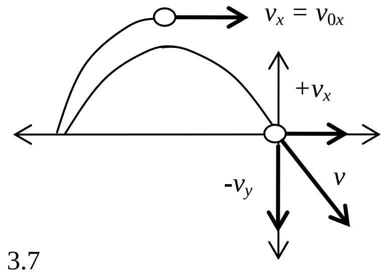


Fig 3.7

$+x$	Always
$-y$	Lands lower than y_0
$y = 0$	Lands same height as y_0
$+y$	Lands higher than y_0

$a_x = 0$ No a in the x-direction

$v_x = v_{0x}$

What is it doing at the end of the problem in the y-direction?

$\boxed{\pm v_y}$

It is usually moving downward at the end of the problem.

So v_y is ***<u>usually negative</u>***

The final $\mathbf{v}$ must be resolved.

$v = \sqrt{v_x^{\ 2} + v_y^{\ 2}}$

If $y = y_0$

$v_y = -v_{0y}$ & $v = v_0$

Projectile Motion Strategies

1. ***Horizontal Launch:*** Since $v_{0y} = 0$ and $v_{0x} = v_0$, then use $\boxed{y = 1/2\ gt^2}$ and $\boxed{x = v_{0x}t}$.
2. ***When time (t) or range (x) is given:*** Start with $\boxed{x = v_{0x}t}$ and then $\boxed{y = y_0 + v_{0y}t + 1/2\ gt^2}$.
3. ***No x and no t:*** Time is the key to falling body & projectile problems. Two strategies are useful when time is missing.

1st $y = y_0 + v_{0y}t + 1/2\ gt^2$	1st $\boxed{v_y^{\ 2} = v_{0y}^{\ 2} + 2g\left(y - y_0\right)}$ solve for v_y which is usually ***<u>negative</u>***.
2nd Quadratic Equation	2nd $\boxed{v_y = v_{0y} + gt}$ use $-v_y$ from above to get t.
3rd $\boxed{x = v_{0x}t}$	3rd $\boxed{x = v_{0x}t}$ use t from above to solve for range x.

Other Projectile Motion Facts

Any ***two launch angles that add to 90°*** will arrive at the same landing site if fired on level ground. Examples: 15° and 75°, 30° and 60°, and 40° and 50°. ***Maximum range*** (maximum distance in the x-direction) is achieved by launching at 45° (45° + 45° = 90°). ***Maximum altitude*** (maximum distance in the y-direction) is achieved by firing straight up, at 90°.

Circular Motion

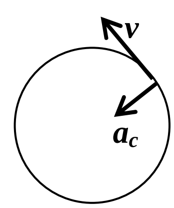


Fig 3.8

Frequency: How often a repeating event happens. Measured in revolutions per second.

Period: The time for one revolution, $\boxed{T = \frac{1}{f}}$. Time is in the numerator.

Velocity: In uniform circular motion the magnitude (speed) of the object is not changing. However, the direction is constantly changing, and this means a change in velocity (a vector composed of both magnitude and direction). In circular motion one can describe the rate of motion as either a speed or as a **tangential velocity** $\boxed{v = \frac{2\pi r}{T}}$. This velocity is an instantaneous velocity and it is directed tangent to the curve.

Centripetal Acceleration: The object is continually turning toward the center of the circle, but never gets there due to its tangential velocity. This centripetal (center seeking) change in velocity, is a centripetal acceleration $\boxed{a_c = \frac{v^2}{r}}$

AP Physics C

Practice Problems: “Vectors”

Multiple Choice Questions

1. The components of vector $\vec{A}$ are given as follows:

   Ax = 10.5   Ay = 15.2

   What is the magnitude of the vector?

   A. 10.5   B. 15.2   C. 18.5   D. 25.7   E. 4.7

2. The components of vector $\vec{A}$ are given as follows:

   Ax = 5.6   Ay = -4.7

   What is the angle between vector $\vec{A}$ and positive direction of x – axis?

   A. 320˚   B. 180˚   C. 90˚   D. 127˚   E. 230˚

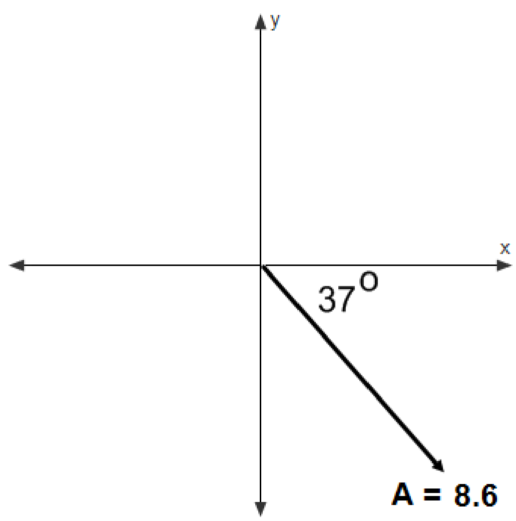
3. The components of vectors $\vec{A}$ and $\overrightarrow{B}$ are given as follows:

   Ax = 5.1   Bx = -2.6

   Ay = -5   By = -4.3

   What is the magnitude of vector sum $\vec{A} + \vec{B}$

   A. 5.1   B. 2.5   C. -9.3   D. 9.6   E. -3.8



4. The magnitude of vector $\vec{A}$ is 8.6. Vector lies in the fourth quadrant and forms an angle of 37˚ with the x-axis. What are the components of vector $\vec{A}$?

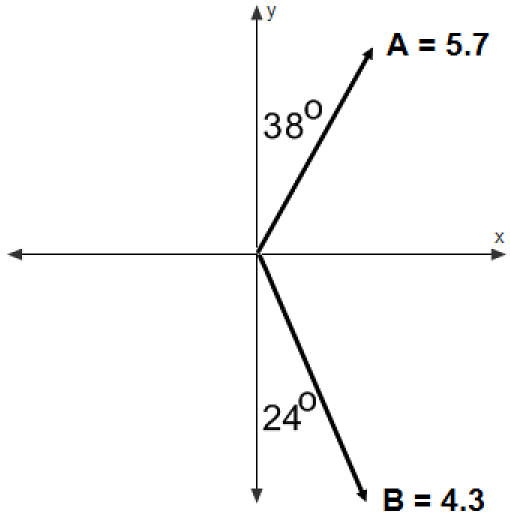
   A. Ax = 8.6   Ay = -8.6

   B. Ax = -6.9   Ay = 5.2

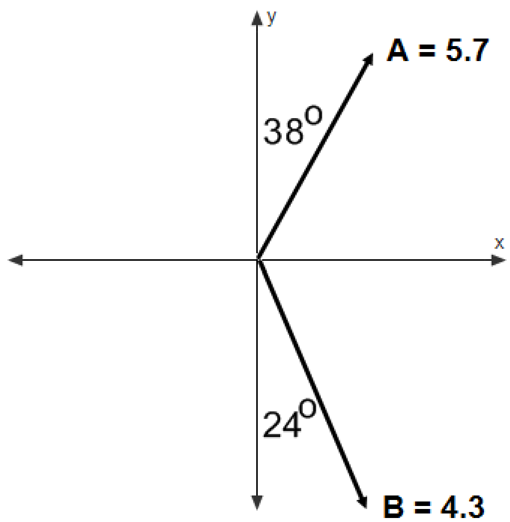
   C. Ax = -6.9   Ay = -5.2

   D. Ax = 6.9   Ay = 5.2

   E. Ax = 6.9   Ay = -5.2



5. Find the magnitude of vector $\vec{C} = \vec{A} - \vec{B}$. Use all the information presented by the graph.
   A. 5.7 B. 6.9 C. 7.4 D. 8.6 E. 9.7



6. Find the dot product of two vectors $\vec{A} \cdot \vec{B}$. Use all the information presented by the graph.
   A. 8.6 B. 3.5 C. -11.6 D. -17.5 E. 9.4

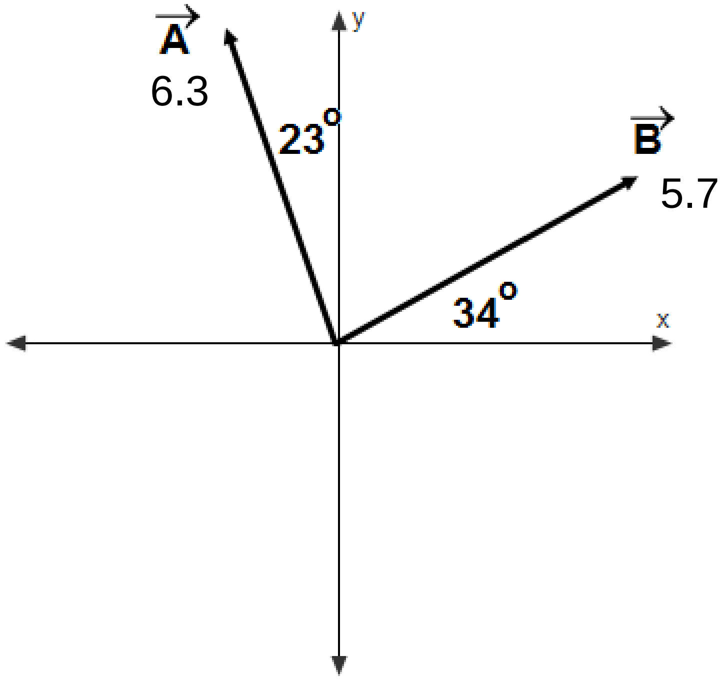
7. Two vectors are given as follows:

   $\vec{A}$ = -2i – 5j + 2k $\vec{B}$ = -4i – 2j – 3k

   What is the angle between the vectors?

   A. 114° B. 67° C. 41° D. 132° E. 94°

Vectors $\vec{A}$ and $\vec{B}$ are shown. Vector $\vec{C}$ is given by $\vec{C} = B$ - $\vec{A}$. Please refer to this figure for problems 8-9.



8. The magnitude of $\vec{C}$ is closest to
   a) 3.9 b) 5.9 c) 6.8 d) 7.7 e) 8.4

9. The angle, measured from the x-axis to vector $\vec{C}$, in degrees, is closest to:
   a) 20˚ b) 34˚ c) 67˚ d) 70˚ e) 82˚

10. The components of vector $\vec{Z}$ are given as follows:
    $Z_x = 10.7$ $Z_y = 8.3$

    What is the magnitude of the vector?

    a) 7.8 b) 9.5 c) 14.2 d) 16 e) 13.6

11. The components of vector $\vec{Q}$ are given as follow:
    $Q_x = 23.5$ $Q_y = 18.6$
    What is the measure of the angle, in degrees, that the resultant vector makes with the x-axis?
    a) 38.4˚ b) 47.9˚ c) 56.3˚ d) 62˚ e) 74.7˚

12. The components of vectors $\vec{U}$ and $\vec{V}$ are given as follow:

$U_x = -8.6$ $V_x = 10.7$

$U_y = 9.4$ $V_y = 4.1$

What is the magnitude of the vector sum $\vec{U} + \vec{V}$?

a) 9.8 b) 13.7 c) 14.6 d) 15.3 e) 16.9

13. Which of the following statements is true?

a) A scalar quantity can be added to a vector

b) It is possible for the magnitude of a vector to equal zero even though one of its components is non-zero

c) Scalar quantities are path dependent, while vectors are not.

d) Scalar quantities and vector quantities can both be added algebraically

e) A scalar contains magnitude and direction while a vector does not.

Questions 14-16:

Two vectors are given as follows:

$\vec{A} = -3\vec{\imath} + 6\vec{\jmath} - 5\vec{k}$

$\vec{B} = -2\vec{\imath} + 3\vec{\jmath} + \vec{k}$

14. The vector dot product $\vec{A} \cdot \vec{B}$ equals:

a) -12 b) 10 c) 14 d) 19 e) 20

15. The difference between vectors $\vec{A}$ and $\vec{B}$ is:

a) $-\vec{\imath} + 9\vec{\jmath} - 4\vec{k}$

b) $-\vec{\imath} + 3\vec{\jmath} - 6\vec{k}$

c) $-3\vec{\imath} + 3\vec{\jmath} - 6\vec{k}$

c)$-5\vec{\imath} + 9\vec{\jmath} - 4\vec{k}$

d) $-6\vec{\imath} + 18\vec{\jmath} - 5\vec{k}$

16. The magnitude of the sum of the vectors $\vec{A}$ and $\vec{B}$ is most nearly:

a) 6.8 b) 7.4 c) 9.0 d) 10.4 e) 11

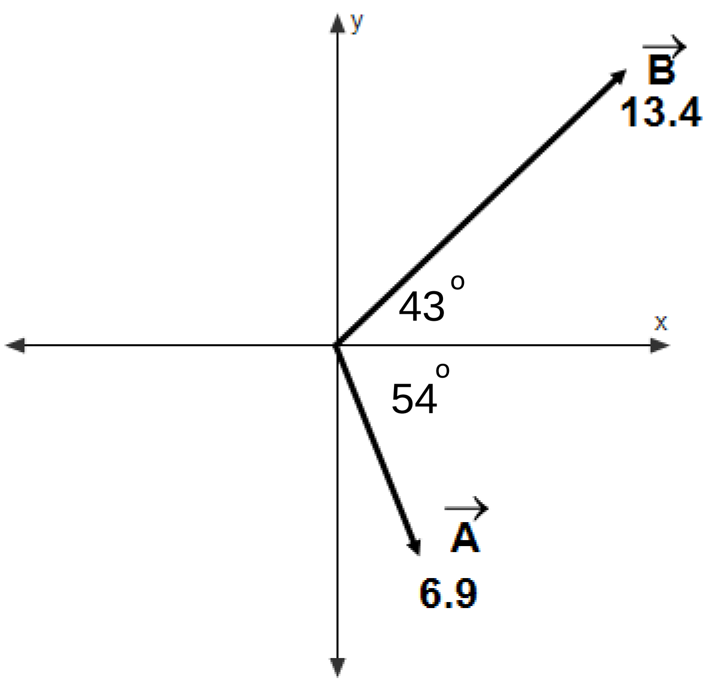
17. The components of vector $\vec{E}$ are as follows:

$E_x = -34.8$ $E_y = -23.6$

What is the measure of the angle, in degrees, formed by vector $\vec{E}$ and +x-axis?

a) -145.9 b)214.1 c) 34.1 d) 145.9 e) 195.7

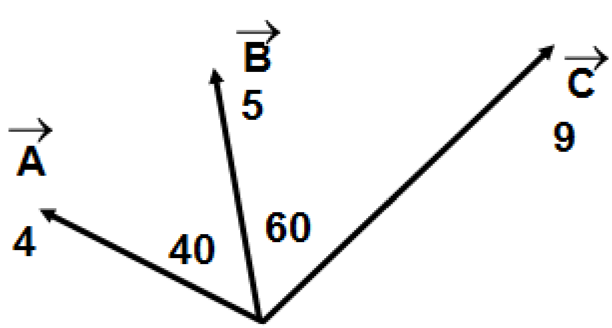
Vectors $\vec{A}$ and $\vec{B}$ are shown. Vector $\vec{C}$ is given by $\vec{C} = \vec{A} + \vec{B}$. Refer to this figure for problems 18-19.



18. What is the magnitude of vector $\vec{C}$?

a) 4.7 b) 11.9 c) 14.3 d) 16.7 e) 17.2

Three Vectors are given as shown. Refer to this figure for numbers 19-21.



19. In the figure above, the magnitude and direction of the vector product $\vec{A} \times \vec{B}$ are closest to:

a) 20, directed out of the plane.
b) 20, directed into the plane.
c) 13, directed out of the plane.
d) 13, directed into the plane.
e) 13, directed on the plane.

20. The magnitude and direction of the vector product $\vec{C} \times \vec{B}$ are closest to

a) 23, directed into the page.
b) 23, directed out of the page.
c) 23, directed on the plane.
d) 39, directed into the page.
e) 39, directed out of the page.

21. The scalar dot product of $\vec{A} \cdot \vec{B}$ is closest to:

a) 15 b) 10 c) 17 d) 21 e) 25

22. Two vectors are given as follows:

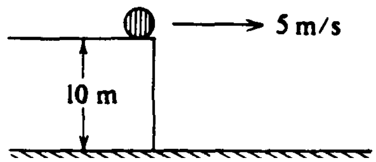
$\vec{A}$ = -2i – 5j + 2k $\vec{B}$ = -5i – 2j – 3k

Find the magnitude of the following vector: $\vec{A} \times \vec{B}$.

A. 12 B. 43 C. 18 D. 26 E. 31

Answer Key.

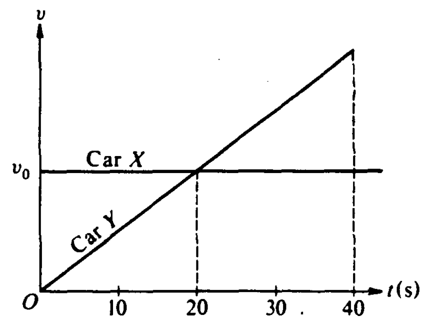
1. C
2. A
3. D
4. E
5. D
6. C
7. B
8. D
9. A
10. E
11. A
12. B
13. C
14. D
15. B
16. E
17. A
18. C
19. D
20. E
21. A
22. E



1 An object slides off a roof 10 meters above the ground with an initial horizontal speed of 5 meters per second as shown above. The time between the object's leaving the roof and hitting the ground is most nearly

(A) $\frac{1}{2}$ s (B) $\frac{1}{\sqrt{2}}$ s (C) $\sqrt{2}$ s (D) 2 s (E) $5\sqrt{2}$ s

<u>Questions 2-3</u>



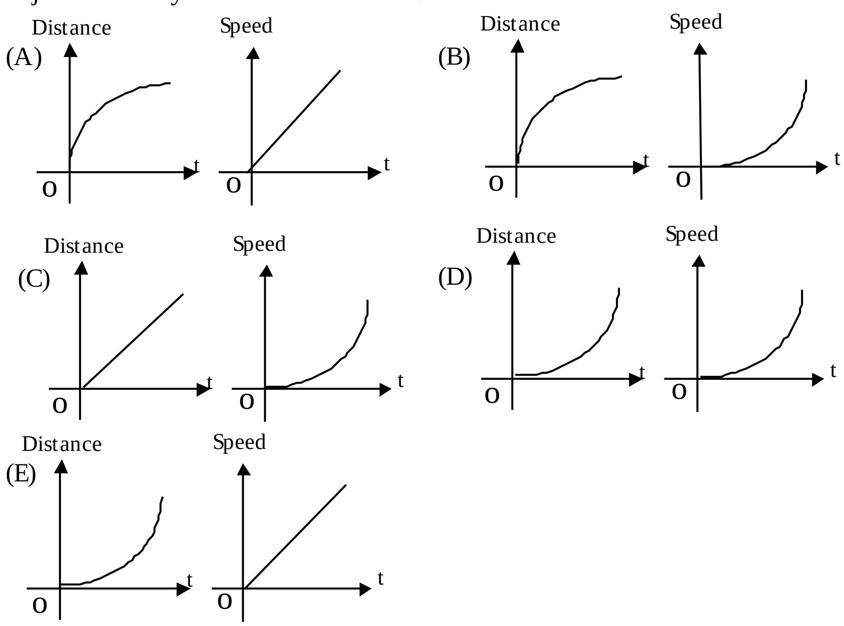
At time t = 0, car X traveling with speed v0 passes car Y. which is just starting to move. Both cars then travel on two parallel lanes of the same straight road. The graphs of speed v versus time t for both cars are shown above.

2 Which of the following is true at time t = 20 seconds?
(A) Car Y is behind car X. (B) Car Y is passing car X. (C) Car Y is in front of car X.
(D) Both cars have the same acceleration. (E) Car X is accelerating faster than car Y.

3 From time t = 0 to time t = 40 seconds, the areas under both curves are equal. Therefore, which of the following is true at time t = 40 seconds?
(A) Car Y is behind car X. (B) Car Y is passing car X. (C) Car Y is in front of car X.
(D) Both cars have the same acceleration. (E) Car X is accelerating faster than car Y.

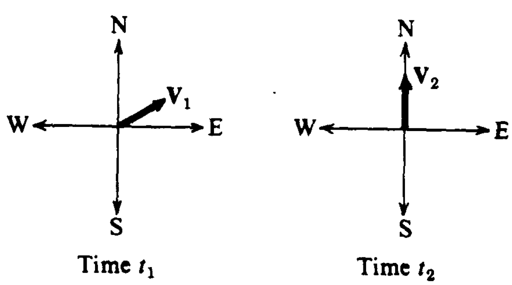
4 A body moving in the positive x direction passes the origin at time t = 0. Between t = 0 and t = 1 second, the body has a constant speed of 24 meters per second. At t = 1 second, the body is given a constant acceleration of 6 meters per second squared in the negative x direction. The position x of the body at t = 11 seconds is
(A) +99 m (B) +36 m (C) -36 m (D) -75 m (E) -99 m

5 Which of the following pairs of graphs shows the distance traveled versus time and the speed versus time for an object uniformly accelerated from rest?



6 An object released from rest at time t = 0 slides down a frictionless incline a distance of 1 meter during the first second. The distance traveled by the object during the time interval from t = 1 second to t = 2 seconds is
(A) 1 m (B) 2 m (C) 3 m (D) 4m (E) 5 m

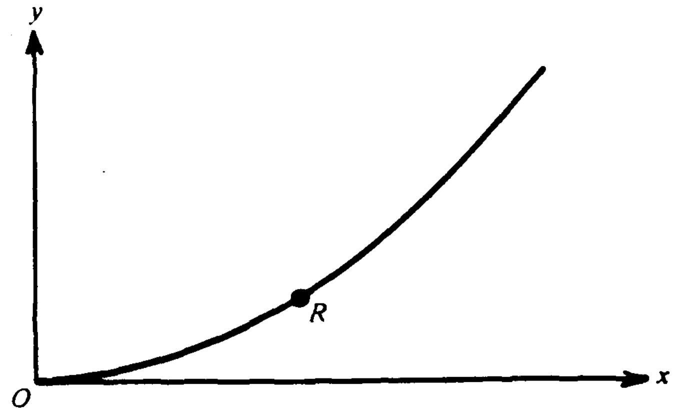
7 Two people are in a boat that is capable of a maximum speed of 5 kilometers per hour in still water, and wish to cross a river 1 kilometer wide to a point directly across from their starting point. If the speed of the water in the river is 5 kilometers per hour, how much time is required for the crossing?
(A) 0.05 hr (B) 0.1 hr (C) 1 hr (D) 10 hr
(E) The point directly across from the starting point cannot be reached under these conditions.



8 Vectors V_1 and V_2 shown above have equal magnitudes. The vectors represent the velocities of an object at times t_1, and t_2, respectively. The average acceleration of the object between time t_1 and t_2 was
(A) zero (B) directed north (C) directed west (D) directed north of east (E) directed north of west

9 A projectile is fired from the surface of the Earth with a speed of 200 meters per second at an angle of 30° above the horizontal. If the ground is level, what is the maximum height reached by the projectile?
(A) 5 m (B) 10 m (C) 500 m (D) 1,000 m (E) 2,000 m

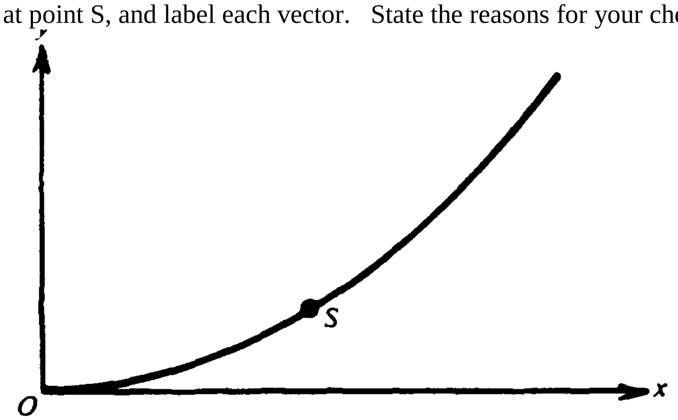
10 A particle moves along the x-axis with a nonconstant acceleration described by a = 12t, where a is in meters per second squared and t is in seconds. If the particle starts from rest so that its speed v and position x are zero when t = 0, where is it located when t = 2 seconds?
(A) x = 12 m (B) x = 16m (C) x = 24 m (D) x = 32 m (E) x = 48 m



1983M1. A particle moves along the parabola with equation $y = \frac{1}{2}x^2$ shown above.

a. Suppose the particle moves so that the x-component of its velocity has the constant value $v_x = C$; that is, $x = Ct$
   i. On the diagram above, indicate the directions of the particle's velocity vector **v** and acceleration vector **a** at point R, and label each vector.
   ii. Determine the y-component of the particle's velocity as a function of x.
   iii. Determine the y-component of the particle's acceleration.

b. Suppose, instead, that the particle moves along the same parabola with a velocity whose x-component is given by $v_x = C/(1+x^2)^{1/2}$
   i. Show that the particle's speed is constant in this case.
   ii. On the diagram below, indicate the directions of the particle's velocity vector **v** and acceleration vector **a** at point S, and label each vector. State the reasons for your choices.



Applications of the Derivative

The most common application of the derivative that we use in AP Physics is the "position – velocity - acceleration" application. (By the way, in AP Physics we use "**x"** to represent position, not "**d"**, which is what we used this past year.)
Since **velocity** is ***the slope of a position vs time*** and **acceleration** is the ***slope of velocity versus time***, then we can say (in "calculus-speak") that *velocity is the derivative of position* and *acceleration is the derivative of velocity*.

There are a bunch of other uses for the derivative in AP Physics, but we'll get to them during the school year. So, for the time being,

Given: **x(t)** **position** as a function of **time**

Then: **dx/dt = velocity** **and** **dv/dt = acceleration**

> Here's a tip that may help you to remember whether or not to use a derivative:
>
> If a quantity can be expressed as a *quotient* in its "basic" form, then that same quantity can be found by taking a *derivative*.
>
> For example, **v = x/t** and **I = q/t** (basically)
>
> Then, in the world of calculus, **v = dx/dt** and **I = dq/dt**

Example 1

A particle's position is given by: $x = 4t^3 - 2t$, where x is in meters and t is in seconds.

(a) Find the velocity of the particle at time t = 3 seconds.

(b) Find the time at which the particle stops momentarily

(c) Find the particle's acceleration at time t = 6 seconds.

Solution:

$v = dx/dt = 12t^2 - 2$ and $a = dv/dt = 24t$

(a) $\therefore v_3 = 12(3)^2 - 2 = 106$ m/s

(b) if v = 0, then $0 = 12t^2 - 2$

$\therefore 2 = 12t^2$

$\therefore t^2 = 1/6$

$\therefore t = 0.41$ seconds

(c) $a_6 = 24(6) = 144$ m/s^2

Problem Set 1

1. The distance in meters that a particle can move in time t seconds is given by: $\mathbf{x = 180t - 5t^2}$. At what time does the particle stop moving?

2. A particle projected vertically reaches an elevation, in meters, given by: $\mathbf{y = 50t - 5t^2}$, where t is in seconds. How fast is the particle moving when it reaches an elevation of 75 meters?

3. A particle's position in meters is given by : $\mathbf{x = t^3 - 3t}$, where t is in seconds.
   (a) Find the particle's velocity at time 5 seconds.
   (b) Find the particle's acceleration at time 2 seconds.

4. A particle's position in meters is given by: $\mathbf{x = 2t^3 - 4t}$, where t is in seconds. Find the particle's acceleration at each instant for which the speed is zero.

5. A particle's position is given by: $\mathbf{x = 3t - t^3}$
Find the following:

(a) The *average* speed between t = 0 second and t = 2 seconds.
(**Note**: $v_{avg} = \Delta x/\Delta t$… *always*, for *any* motion)

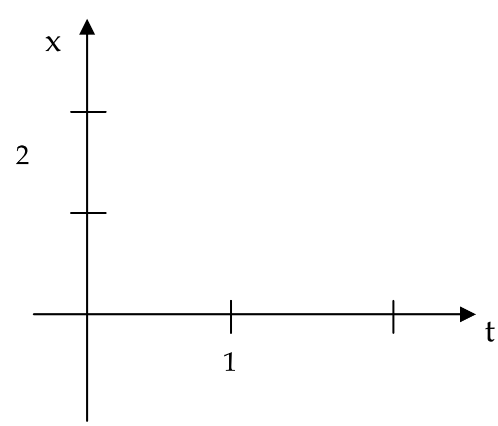
(b) The instantaneous *velocity* at t = 2 seconds

(c) The instantaneous *speed* at t = 3 seconds

(d) The *average* acceleration between t = 0 seconds and t = 2 seconds
(**Note:** $a_{avg} = \Delta v/\Delta t$… *always*, for *any* motion)

(e) The *instantaneous* acceleration at t = 2 seconds

(f) *Sketch* a graph of x vs t by doing the following:



- Find x when t = 0 (where the graph starts)
- Find t when x = 0 (where x crosses the axis)
- Find where the slope (the derivative!) = 0

(this tells you where the graph “turns around”)

PhysicsJedi.net

AP Physics C
Essential Calculus

Examples of the calculus skills identified on the College Board AP Physics C Equation Sheet.

<u>Differentiation - Power Rule</u>

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

<u>Example #1</u>

Given the function, $y = y_o + v_o t + \frac{1}{2}gt^2$

Find the first derivative, $\frac{d}{dt}(y)$.

$$\frac{dy}{dt} = \frac{dy_o}{dt} + \frac{d\left(v_o t\right)}{dt} + \frac{d\left(1/2\, gt^2\right)}{dt}$$

$$\frac{dy}{dt} = 0 + v_o + gt$$

$$\frac{dy}{dt} = v_o + gt$$

$$v = v_o + gt$$

Find the second derivative, $\frac{d}{dt}(v)$.

$$\frac{dv}{dt} = \frac{dv_o}{dt} + \frac{d(gt)}{dt}$$

$$\frac{dv}{dt} = 0 + g$$

$$\frac{dv}{dt} = g$$

$$a = g$$

<u>Example #2</u>

$$\frac{d}{dx}\left(u^n\right) = nu^{n-1}\frac{du}{dx}$$

Given $v = \sqrt{2ax} = (2ax)^{\frac{1}{2}} = u^{\frac{1}{2}}$; $u = 2ax$; $\frac{du}{dx} = 2a$

$$\frac{d}{dx}\left(u^n\right) = \frac{d}{dx}\left(u^{\frac{1}{2}}\right) = \frac{1}{2}u^{-\frac{1}{2}}(2a) = \frac{a}{\sqrt{u}} = \frac{a}{\sqrt{2ax}} = \sqrt{\frac{a}{2x}}$$

Differentiation – Chain Rule

$$\frac{df}{dx} = \frac{df}{du}\frac{du}{dx}$$

Example #3

Given the functions $v_x = at$ (1) and $x = \frac{1}{2}at^2$ (2). Find the acceleration, $a = \frac{dv}{dt}$. From the equations given it is obvious the acceleration is "a".

Now write an equation for the velocity as a function of horizontal position. Solve equation (2) for time and substitute into equation (1).

$$v_x = a\left(\sqrt{\frac{2x}{a}}\right) = \sqrt{2ax}$$

Again find the acceleration, $a = \frac{dv}{dt}$. Since v_x is not a function of time, apply the chain rule.

$$a = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = \frac{dv}{dx}v$$

$$\frac{dv}{dx} = \frac{1}{2}(2ax)^{-\frac{1}{2}}2a = \frac{a}{\sqrt{2ax}}$$

$$a = \frac{dv}{dx}v = \left(\frac{a}{\sqrt{2ax}}\right)\sqrt{2ax} = a$$

Differentiation – Trig Functions

$$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\cos x) = -\sin x \qquad \frac{d}{dx}(\tan x) = \sec^2 x$$

The derivatives of ALL trig functions starting with "c" are negative.

$$\frac{d}{dx}(\sin u) = \cos u\frac{du}{dx} \qquad \frac{d}{dx}(\cos u) = -\sin u\frac{du}{dx}$$

Example

Find the first derivative $\frac{d}{dt}(A\sin(\omega t + \phi))$

Let $u = \omega t + \phi$ and $\frac{du}{dt} = \omega$

$$\frac{d}{dt}(\sin u) = A\frac{d}{dt}(\sin(A\omega + \phi)) = A\cos(A\omega + \phi)(\omega) = A\omega\cos(A\omega + \phi)$$

Differentiation – The Product Rule

Always use the product rule; never the quotient rule.

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

Example

Differentiate $f = \frac{x}{\sqrt{x^2+1}}$

Express f as product; $f = x(x^2+1)^{-\frac{1}{2}}$

Let $u = x$ and $v = (x^2+1)^{-\frac{1}{2}}$

$$\frac{d}{dx}(f) = \frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{d}{dx}(f) = \frac{d}{dx}(uv) = x\left(-\frac{1}{2}\right)(x^2+1)^{-\frac{3}{2}}(2x) + (x^2+1)^{-\frac{1}{2}}$$

$$\frac{d}{dx}(f) = \frac{d}{dx}(uv) = -\frac{x^2}{(x^2+1)^{\frac{3}{2}}} + \frac{1}{(x^2+1)^{\frac{1}{2}}}$$

$$\frac{d}{dx}(f) = \frac{d}{dx}(uv) = -\frac{x^2}{(x^2+1)^{\frac{3}{2}}} + \frac{(x^2+1)}{(x^2+1)^{\frac{3}{2}}}$$

$$\frac{d}{dx}(f) = \frac{d}{dx}(uv) = \frac{1}{(x^2+1)^{\frac{3}{2}}}$$

Differentiation – e

$$\frac{d}{dx}\left(e^{x}\right) = e^{x}$$

$$\frac{d}{dx}\left(e^{u}\right) = e^{u}\frac{du}{dx}$$

Example

Differentiate e^{-kx} Let $u = -kx$ and $\frac{du}{dx} = -k$

$$\frac{d}{dx}\left(e^{-kx}\right) = e^{u}\frac{du}{dx}$$

$$\frac{d}{dx}\left(e^{-kx}\right) = e^{-kx}(-k)$$

$$\frac{d}{dx}\left(e^{-kx}\right) = -ke^{-kx}$$

Differentiation – ln

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\frac{d}{dx}(\ln u) = \frac{1}{u}\frac{du}{dx}$$

Integration – Power Rule

$$\int x^n dx = \frac{1}{n+1}x^{n+1}, n \neq -1$$

Example #1

$$\int kx^2 dx = \frac{1}{2+1}kx^{2+1} = \frac{1}{3}kx^3$$ This is an indefinite integral.

Problems typically require limits of integration.

$$\int_2^3 kx^2 dx = \frac{1}{3}kx^3 \Big|_2^3 = \frac{1}{3}k3^3 - \frac{1}{3}k2^3 = 9k - \frac{8}{3}k = \frac{19}{3}k$$

Example #2

$$\int u^n du = \frac{1}{n+1}u^{n+1}, n \neq -1$$

$$\int \frac{2x}{(x^2+1)^3}dx$$ Let $u = x^2 + 1$ and $du = 2xdx$

$$\int \frac{2x}{(x^2+1)^3}dx = \frac{1}{-3+1}(x^2+1)^{-3+1} = -\frac{1}{2}(x^2+1)^{-2} = \frac{-1}{2(x^2+1)^2}$$

Example #3

$$\int \frac{x}{(x^2+1)^3}dx$$ Let $u = x^2 + 1$ and $du = 2xdx$

From the equation du = xdx; need du = 2xdx; missing 2.   Multiple by “1”.

$$\int \frac{x}{(x^2+1)^3}\frac{2}{2}dx = \frac{1}{2}\int \frac{2x}{(x^2+1)^3}dx = \frac{1}{2}\left(\frac{-1}{2(x^2+1)^2}\right) = \frac{-1}{4(x^2+1)^2}$$

Integration – Trig Functions

$$\int \sin x = -\cos x$$

$$\int \cos x = \sin x$$

Integration – e

$$\int e^x = e^x$$

$$\int e^u du = e^u$$

Example #1

$\int -2kxe^{-kx^2}dx$ Let $u = -kx^2$ and $du = -2kxdx$

$$\int -2kxe^{-kx^2}dx = \int e^u du = e^{-kx^2}$$

Example #2

$\int xe^{-kx^2}dx$ Let $u = -kx^2$ and $du = -2kxdx$

From the equation du = xdx; need du = -2kxdx; missing -2k. Multiple by "1".

$$\int xe^{-kx^2}\frac{-2k}{-2k}dx = \frac{1}{-2k}\int -2kxe^{-kx^2}dx = -\frac{1}{2k}e^{-kx^2}$$

Integration – ln

$$\int \frac{1}{x}dx = \ln|x|$$

Example #1

Integrate $\int \frac{dv}{v}$ where "v" represents velocity

$\int \frac{dv}{v} = \ln|v|$ This is an indefinite integral.

Problems typically require limits of integration.

$$\int_{v_i}^{v_f} \frac{dv}{v} = \ln v \,|_{v_i}^{v_f} = \ln v_f - \ln v_i = \ln\left(\frac{v_f}{v_i}\right)$$

Recall $e^{\ln x} = x$

$$\int \frac{du}{u} = \ln|u|$$

Example #2

Integrate $\int \frac{kdq}{(1+kq)}$ where "q" represents charge

Let $u = 1+q$ and $du = kdq$

$$\int \frac{kdq}{(1+kq)} = \int \frac{du}{u} = \ln|1+q|$$

Example #3

$\int \frac{dq}{(k-q)}$ Let $u = 1-q$ and $du = -dq$

From the equation du = dq; need du = -dq; missing -1. Multiple by "1".

$\int \frac{dq}{(k-q)}\frac{-1}{-1} = -1\int \frac{(-1)dq}{k-q} = -\ln|k-q|$ This is an indefinite integral.

Problems typically require limits of integration.

$$\int_0^{q_f} \frac{dq}{(k-q)} = -\ln|k-q| \,|_0^{q_f} = -\ln(k-q_f) - (-\ln(k-0)) = -[\ln(k-q_f) - \ln(k)] = -\ln\left(\frac{k-q}{k}\right)$$

Recall $e^{-\ln x} = e^{\ln\frac{1}{x}} = \frac{1}{x}$

Differential Equations

$$B\frac{d^2f(x)}{dx^2}+Cf(x)=0$$ B and C are constants

Rearrange so the second derivative term has no coefficients.

$$\frac{d^2f(x)}{dx^2}+\frac{C}{B}f(x)=0$$

Suppose the constants "B" and "C" equal 1.

$$\frac{d^2f(x)}{dx^2}+f(x)=0$$

What function when differentiated twice and added to itself equals zero?

$$f(x)=\sin(x) \qquad \frac{d^2f(x)}{dx^2}=-\sin(x)$$

OR

$$f(x)=\cos(x) \qquad \frac{d^2f(x)}{dx^2}=-\cos(x)$$

Now let $B\neq 1$ and $C\neq 1$

Let $f(x)=\sin(\omega x)$ and $\frac{d^2f(x)}{dx^2}=-\omega^2\sin(\omega x)$

Substitute the above equations into the general form of the differential equation.

$$\frac{d^2f(x)}{dx^2}+\frac{C}{B}f(x)=0$$

$$-\omega^2\sin(\omega x)+\frac{C}{B}\sin(\omega x)=0$$

Solve for ω

$$\omega^2=\frac{C}{B} \qquad \omega=\sqrt{\frac{C}{B}}$$

Final form of the solution. $f(x)=\sin\left(\sqrt{\frac{C}{B}}x\right)$ OR $f(x)=A\sin\left(\sqrt{\frac{C}{B}}x\right)$